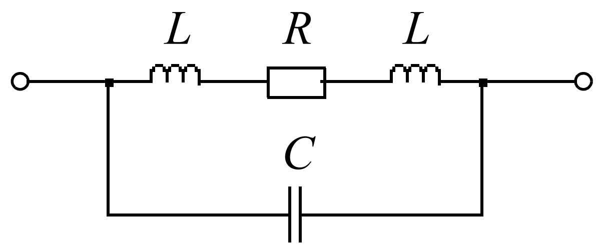
CHAPTER 1

PASSIVE HF COMPONENTS

RESISTORS (1/2)

The equivalent circuit for a resistor in HF is shown in Fig. 1-1. *R* represents the value of the resistor, to which one must add two inductances *L* and a capacitance *C*, representing the parasitic elements needed to model its behavior in HF.



$L \approx 10\ nH$

$C \leq 1\ pF$

Fig 1-1: *Equivalent circuit for a resistor.*

The values of *L* and *C* depend on the way in which this resistor is realized. Three types of resistors are possible:

1) *wirewound* resistors;
2) *carbon-composition* resistors;
3) *metal-film* resistors.

Due to their high inductance, wirewound resistors are primarily used when the dissipated power becomes large (> 10 W), but are not preferred for RF applications. Their impedance corresponds to that of a *resonant parallel circuit*.

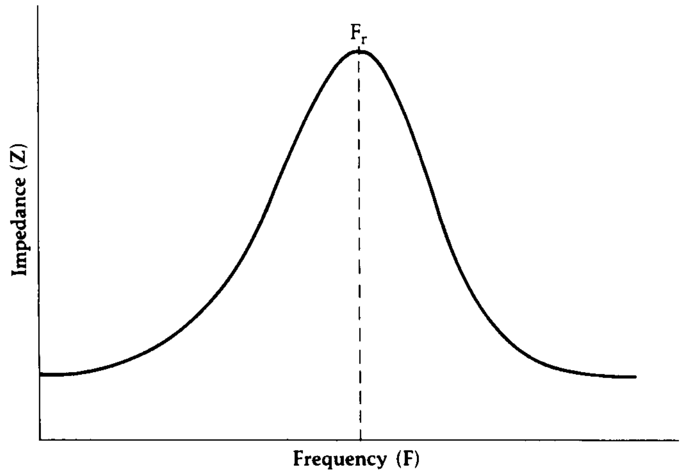


Fig 1-2: *Impedance of a wirewound resistor.*

RESISTORS (2/2)

Carbon-composition resistors are made of many carbon granules immerged in a binding substance. Their HF behavior is *dominated by the capacitance* C, due to all of the parasitic capacitances which exist between each pair of carbon granules.
Metal-film resistors are realized using a thin layer of carbon or a metallic film. As indicated in Fig. 1-3, metal-film resistors have better HF performance than carbon-composition resistors.

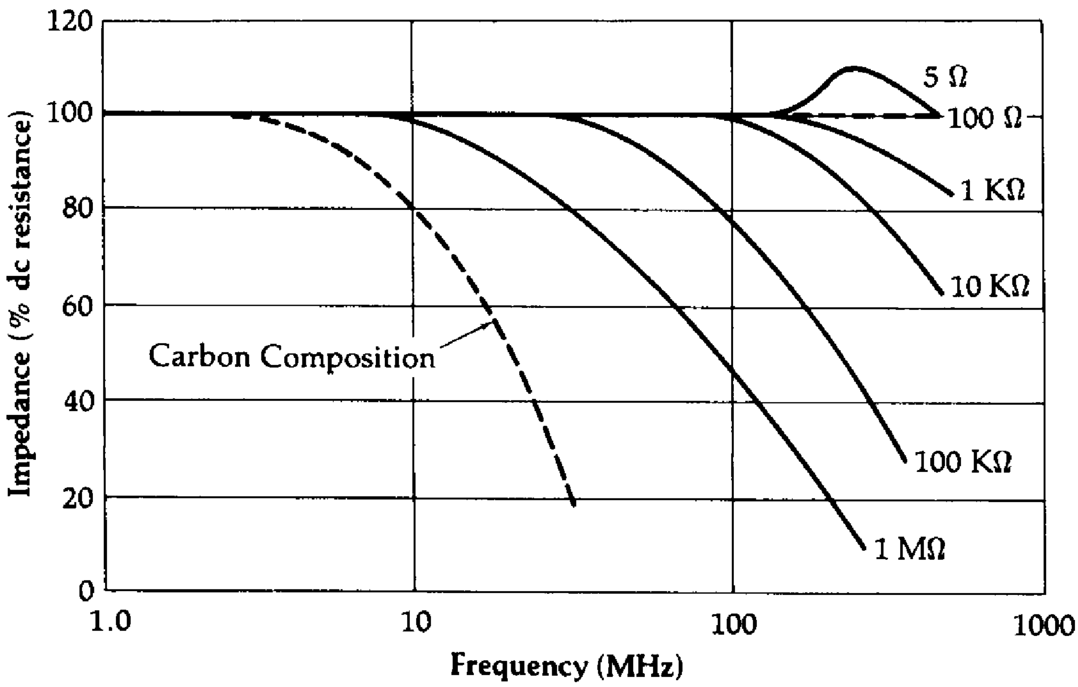


Fig 1-3: *Comparison of impedances of carbon-composition and metal-film resistors.*

For high-resistance resistors, the impedance decreases starting at 10 MHz due to the capacitance, while for small-resistance resistors (< 50 Ω), the inductive effects of the connecting wires and the skin effect become important.
SMD (Surface Mounted Device) technology, used more and more often, has enabled the significant reduction of parasitic effects. These resistors are made of thin films of aluminum or beryllium, and have a small reactance up to frequencies of GHz.

CAPACITORS (1/2)

The capacitor is a frequently-used component for RF. It allows the realization of the following important functions:

1) coupling between amplifier stages;
2) resonant circuits;
3) filters.

A capacitor's characteristics essentially depend on the type of dielectric used. The equivalent circuit for a capacitor is shown in Fig. 1-4, where *C* is the capacitance of the capacitor, *L* is the inductance due to the connections (leads), R_s is the series resistance of those leads, and R_p is the equivalent resistance due to the leakage current and the losses coming from the hysteresis of the dielectric.

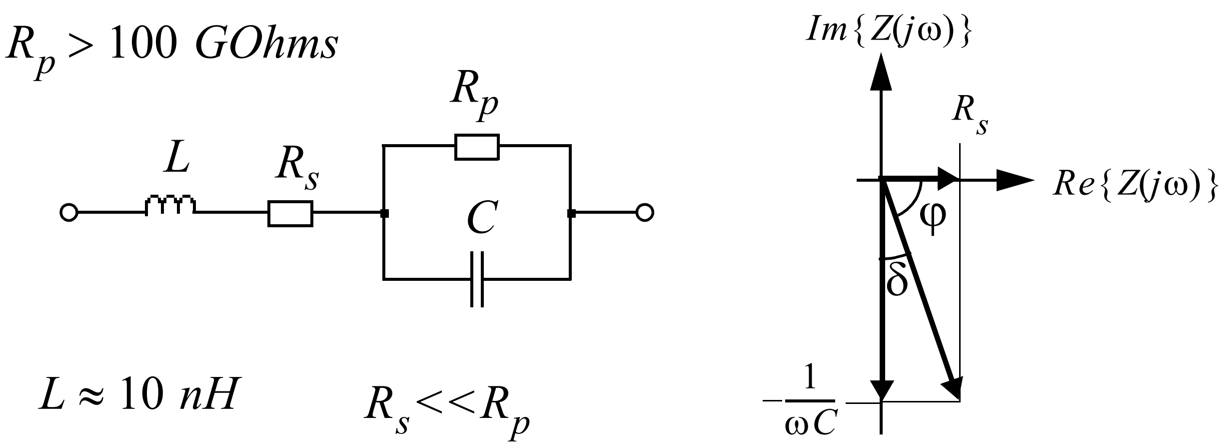


Fig 1-4: *Equivalent circuit of a capacitor.*

The ensemble of these losses in the dielectric and in the connective resistance is summarized by using a *loss factor* $\tan(\delta)$ defined as the ratio between the active dissipated power and the absolute value of the reactive power:

$$\boxed{\tan(\delta) \equiv \frac{P_{active}}{|P_{reactive}|} = \frac{R}{|X|} = \omega R_s C + \frac{1}{\omega R_p C}} \tag{1.1}$$

where $R \equiv Re\{Z(j\omega)\}$ and $X \equiv Im\{Z(j\omega)\}$.

CAPACITORS (2/2)

The inverse of the loss factor is the *quality factor*, given by:

$$\boxed{Q \equiv \frac{1}{\tan(\delta)} = \frac{|P_{reactive}|}{P_{active}} = \frac{|X|}{R} = \left[\frac{1}{Q_s} + \frac{1}{Q_p}\right]^{-1} \cong Q_s} \qquad (1.2)$$

where: $$Q_s \equiv \frac{1}{\omega R_s C} \qquad et: \qquad Q_p \equiv \omega R_p C \qquad (1.3)$$

The impedance corresponding to the diagram in Fig. 1-4 for $\omega << 1/\sqrt{LC}$ and $R_s << R_p$ is therefore given by:

$$Z(j\omega) = R + jX \cong \frac{1}{j\omega C} \cdot \left[1 + j\omega R_s C - \frac{1}{j\omega R_p C}\right] \qquad (1.4)$$

$$= \frac{1}{j\omega C} \cdot \left[1 + \frac{j}{Q}\right] = \frac{1}{j\omega C} \cdot [1 + j\tan(\delta)]$$

The impedance characteristic given by (1.4) is shown in Fig. 1-5. The importance of the lead inductance increases with frequency, until this inductance becomes resonant with the capacitor. Above the resonant frequency F_r, the capacitor acts as an inductor.

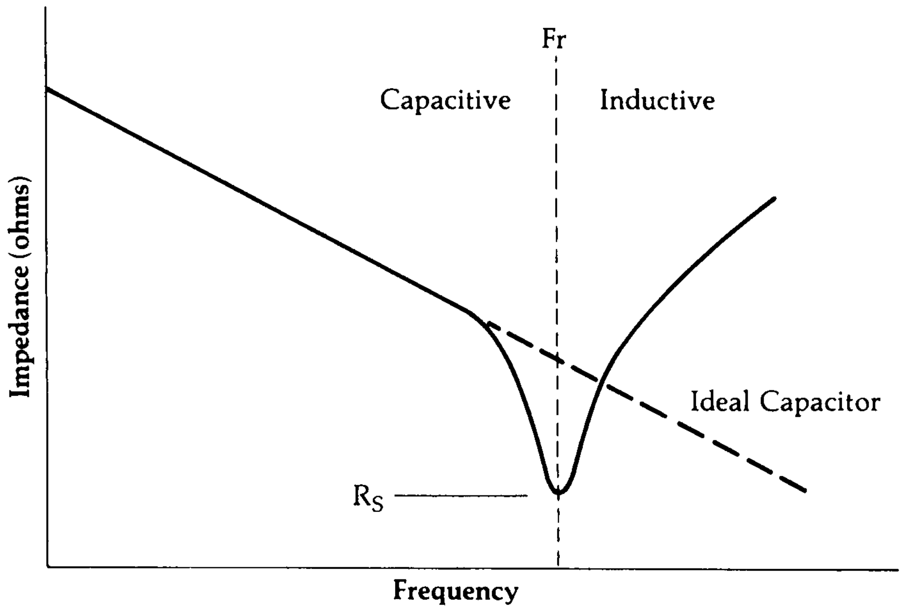


Fig 1-5: *Impedance characteristic of a capacitor.*

CAPACITOR TYPES (1/3)

There are several types of capacitors; in particular:

1) ceramic;
2) mica;
3) metallized-film;
4) SMD;
5) electrolytic.

Ceramic capacitors have relative dielectric constants between 5 and 10'000 and various temperature characteristics. As indicated in Fig. 1-6, the higher the dielectric constant, the more sensitive the capacitor to temperature.

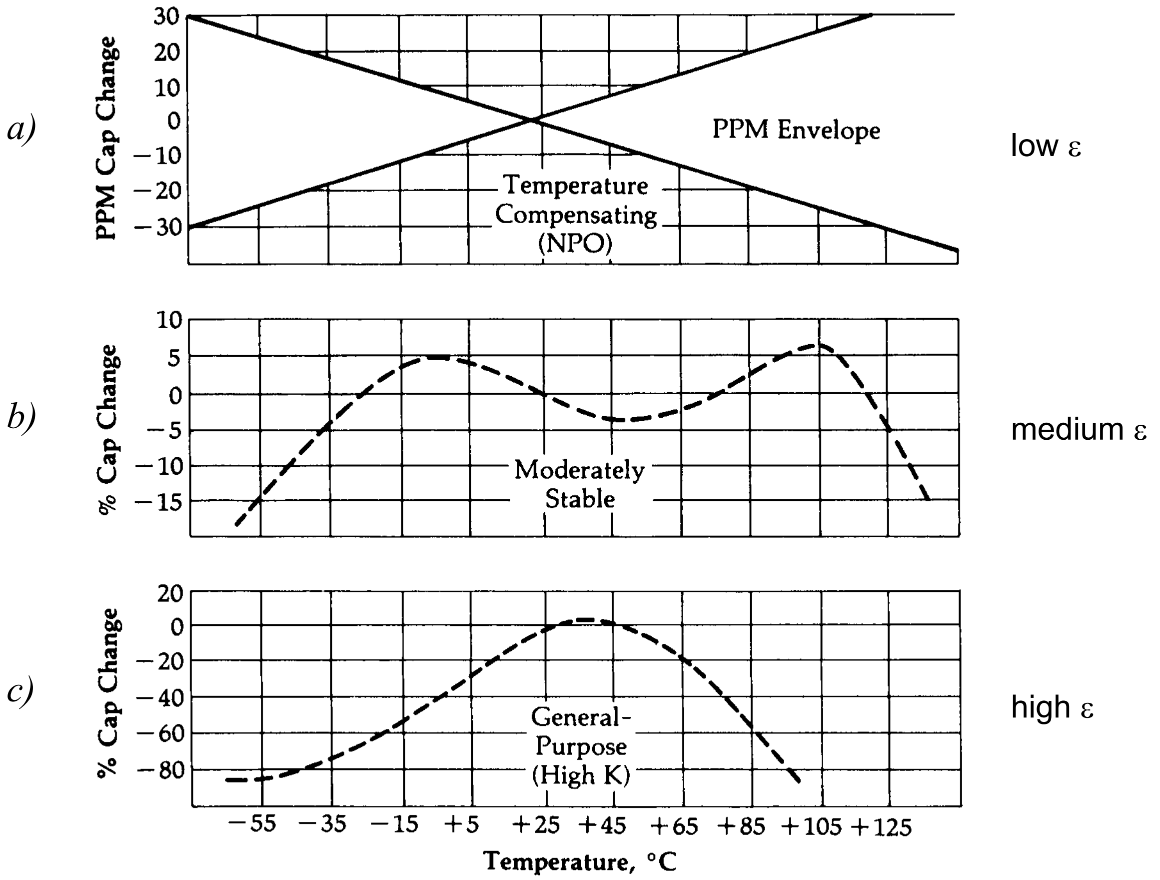


Fig 1-6: *Temperature sensitivity of ceramic capacitors.*

CAPACITOR TYPES (2/3)

Ceramic capacitors with a low dielectric constant (Fig. 1-6 a)) are usually fabricated from two materials with temperature coefficients of opposite signs. This type of capacitor (called NPO: Negative Positive Zero) have thermal coefficients between +150 and –4,700 ppm/°C with tolerances as low as ±15 ppm/°C. These capacitors are primarily used in oscillators.

Moderately stable ceramic capacitors (Fig. 1-6 b)) have a maximum variation of 15% of their nominal value for a temperature range from –60 to +125 °C. In addition, this relative variation is nonlinear. They are primarily used for switching applications. Their advantages over NPO capacitors are their small size and low cost.

Ceramic capacitors with a high dielectric constant (Fig. 1-6 c)) have bad thermal stability and are only used for decouplage applications which require large capacitances.

There are ceramic capacitors available, conceived for HF applications, which have high quality factors. These capacitors are, of course, more expensive.

CAPACITOR TYPES (3/3)

Mica capacitors have a low dielectric constant ($\varepsilon_r \cong 6$), which explains their large size and excellent thermal stability. Silvered mica capacitors have a very low thermal coefficient (usually +20 ppm/°C for a temperature varying between –60°C and +90°C).

Metallized-film capacitors are fabricated using many different dielectrics (teflon, polystyrene, polycarbonate, paper,…). They are available with very tight tolerances (±2%) over the entire usual temperature range. With a larger size and a similar cost to that of ceramic capacitors, they don't offer any particular advantage. In addition, certain metallized-film/polystyrene capacitors have seriously degraded thermal behavior at temperatures above 85 °C.

Finally, *electrolytic capacitors* offer very high capacitances. Unfortunately, due to their design, their parasitic inductances are high, and make them unusable for HF.

INDUCTORS (1/3)

Inductors are made of coils which increase the magnetic flux coupling between each turn. Even though inductors are often used in HF circuits, *the inductor is certainly the component that poses the most problems*. In particular, the impedance changes drastically as a function of frequency. The equivalent circuit for an inductor is shown in Fig. 1-7, where L is the inductance, R_s comes from the series resistance of the coiled wire and the skin effect, and C_d represents the distributed capacitance between the turns of the coil. This capacitance is therefore inversely proportional to the distance between the turns.

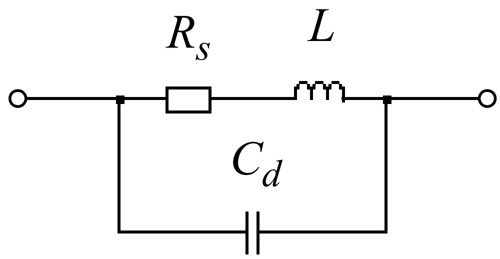


Fig 1-7: *Equivalent circuit of an inductor.*

The corresponding impedance is given by:

$$Z(j\omega) = R + jX = \frac{R_s + j\omega L}{1 - \omega^2 L C_d + j\omega R_s C_d} \cong R_s + j\omega L \qquad (1.5)$$

for $\omega << 1/\sqrt{LC_d}$. As indicated in Fig. 1-8, at low frequency, the inductor acts as an inductance in series with a resistance. As the frequency increases, the impedance deviates from that of an inductance and increases rapidly to reach a maximum value at the resonant frequency $\omega_0 \equiv 1/\sqrt{LC_d}$. Above that point, the impedance decreases and the inductor acts as a capacitor.

INDUCTORS (2/3)

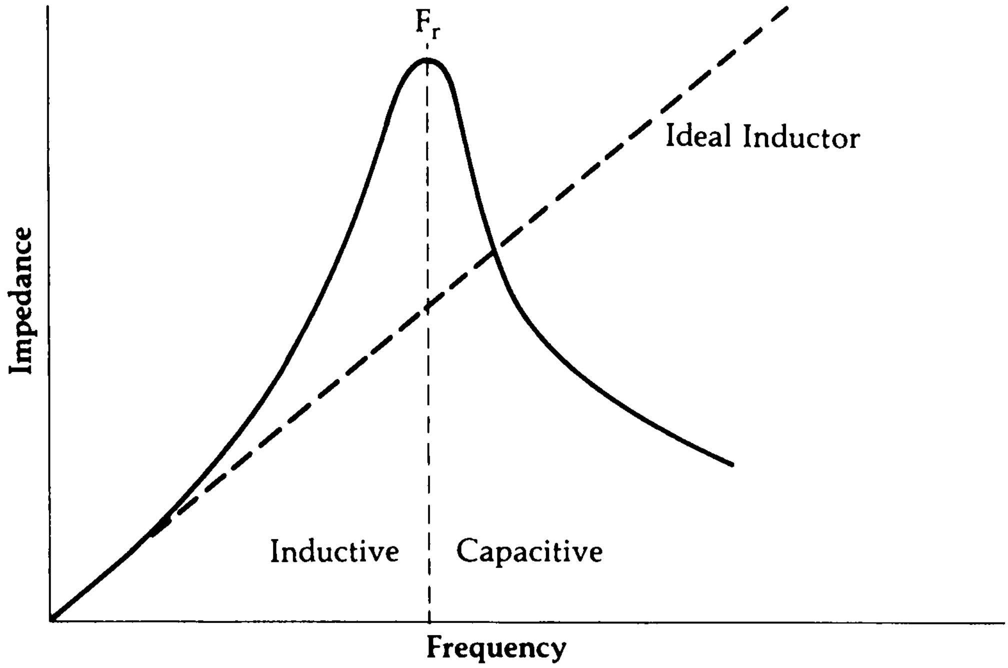


Fig 1-8: *Impedance characteristic of an inductor.*

All of the losses due to series resistance and skin effect are expressed by the *quality factor* Q, defined as the ratio between the absolute value of the reactive power absorbed and the active power dissipated by the impedance $Z(j\omega)$:

$$\boxed{Q \equiv \frac{|P_{reactive}|}{P_{active}} = \frac{|X|}{R} \cong \frac{\omega L}{R_s}} \tag{1.6}$$

For frequencies less than the resonant frequency, the impedance can also be written as:

$$Z(j\omega) = R + jX = jX\left(1 + \frac{R}{jX}\right) \cong j\omega L \cdot \left[1 + \frac{1}{jQ}\right] \tag{1.7}$$

The higher the quality factor, the closer the inductor to an ideal inductor, and the higher the impedance value at resonance ($Z(\omega_0) \cong Q/(\omega_0 C_d)$).

INDUCTORS (3/3)

According to Eqn. 1.6, Q increases proportionally with the frequency. But this result has been reached without considering the skin effect, which tends to increase the series resistance R_s and therefore to reduce the rise of Q as the frequency climbs. As indicated in Fig. 1-9, the quality factor has a maximum, which corresponds to the situation in which the series resistance and the inductor reactance increase at the same rate with frequency. Beyond this maximum, the quality factor decreases rapidly due to the shunt capacitance C_d. The quality factor can be expressed as a function of the real and imaginary parts of Eqn. 1.5:

$$Q \equiv \frac{|X|}{R} \cong \frac{\omega L}{R_s(\omega)} \cdot \left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] \qquad (1.8)$$

Eqn. 1.8 shows that the quality factor is zero when the frequency equals the resonant frequency. This comes from the fact that at resonance, the reactance of the inductance is cancelled out by the reactance of the capacitance.

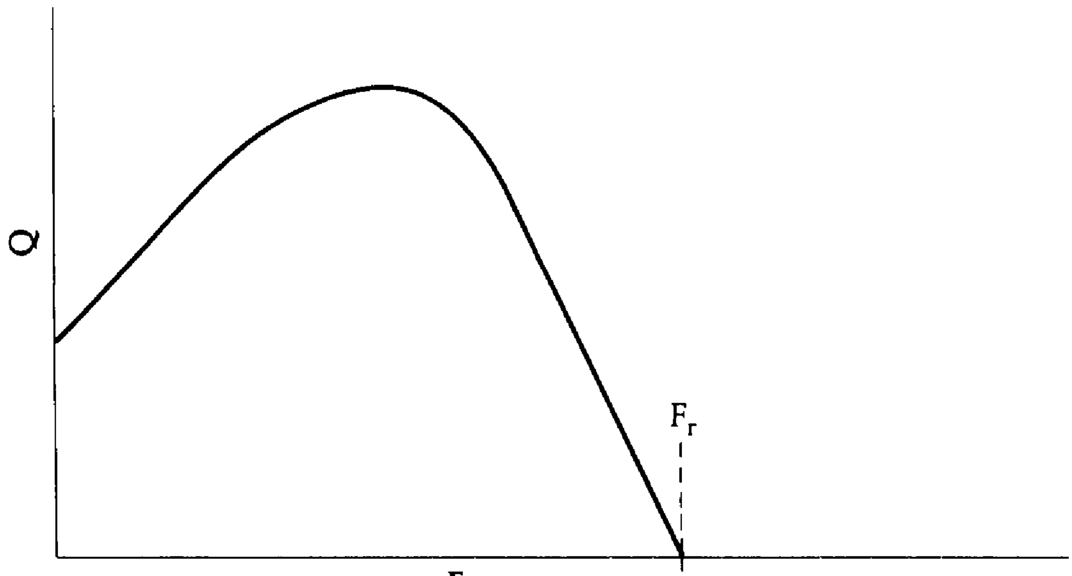


Fig 1-9: *Quality factor as a function of the frequency.*

INCREASING THE QUALITY FACTOR

There are various methods to increase the quality factor and the maximum frequency of use of an inductor. We can name the following possibilities:

1) use a larger-diameter wire, to reduce the series resistance R_s;
2) spread the turns apart, which reduces the coupling capacitance C_d between turns;
3) increase the permeability of the core, by using a magnetic core.

AIR-CORE INDUCTORS

The inductance of a single-layer air-core inductor is approximately given by

$$L = \frac{0.394 \cdot r^2 \cdot N^2}{9r + 10l} \qquad in\ \mu H \tag{1.9}$$

where r is the radius in cm, l the length in cm, N the number of turns, and L the inductance in µH (cf Fig. 1-10). The diameter of the wire is related to the length of the coil by:

$$d \leq \frac{l}{N} \tag{1.10}$$

Note that equation (1.9) is only true for $l > 0.67r$.

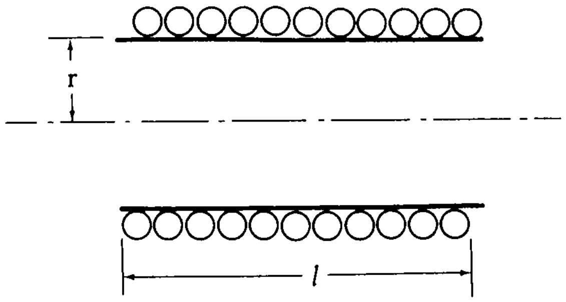


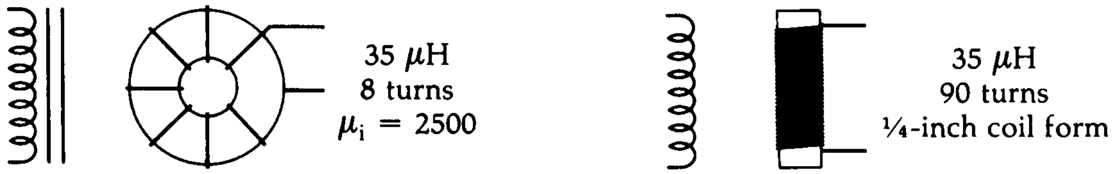
Fig 1-10: *Single-layer air-core inductor.*

Although the quality factor is maximum when the length *l* is equal to the diameter $2r$ of the inductor, in practice, the inductor is usually much longer than its diameter.

To realize an inductor with an inductance L, we choose the length and diameter of the coil, for example, and then calculate the number of turns using Eqn. 1.9. We would then choose the diameter of the wire according to Eqn. 1.10.

MAGNETIC-CORE INDUCTORS

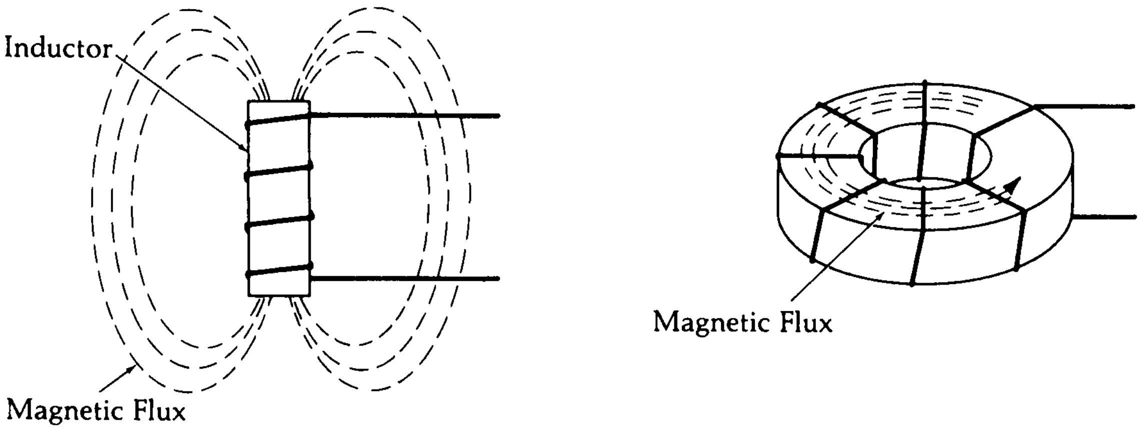
For applications requiring large inductance values and small size, air-core inductors are not appropriate, so it is necessary to use a core with a higher permeability than air. Ferrite cores are generally used. For the same inductance, magnetic-core inductors need less turns and are therefore smaller, while having a larger quality factor (cf Fig. 1-11). They can also have a variable inductance, by inserting the core more or less deeply in the coil.



a) With magnetic core. *b) Without magnetic core.*

Fig 1-11: *Inductor comparison.*

Ferrite cores are most frequently found on the market in the form of a toroid (ring). This form is particularly suitable for the production of high quality factor inductors. The advantage of the toroid over the solenoid is that the tube of magnetic flux is closed on itself, avoiding coupling with neighboring elements (cf Fig. 1-12).

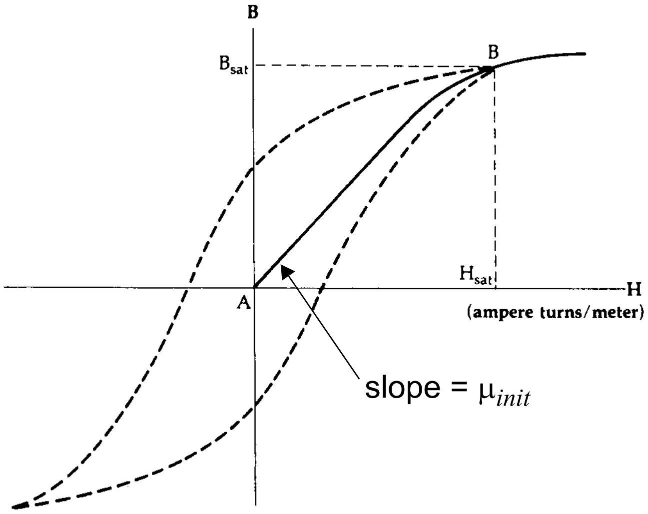


a) Solenoid. *b) Toroid.*

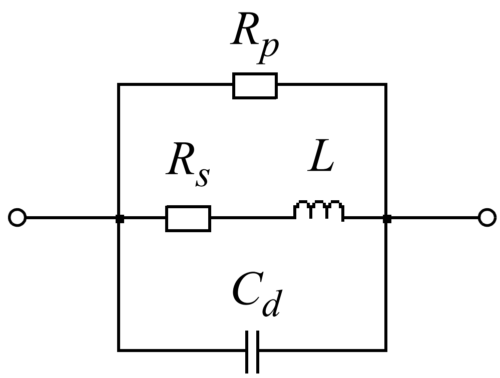
Fig 1-12: *Inductor magnetic flux.*

HYSTERESIS LOSSES (1/2)

Materials such as ferrite give a nonlinear magnetization curve with saturation and hysteresis as shown in Fig. 1-13 a). This hysteresis implies that a change of direction of magnetization requires the absorption of energy. To take these losses into account, we must add a resistance R_p to Fig. 1-7, connected in parallel with the inductance (cf Fig. 1-13 b)).



a) Magnetization curve.



b) Equivalent circuit.

Fig 1-13: *Magnetization curve and equivalent circuit for a magnetic-core inductor.*

For $\omega << 1/\sqrt{LC}$, we can ignore the capacitance C_d, and considering that $R_s << R_p$, the impedance is approximately given by:

$$Z(j\omega) = R + jX \cong R_s + \frac{(\omega L)^2}{R_p} + j\omega L = j\omega L \cdot \left[1 + \frac{1}{jQ_{eq}}\right] \quad (1.11)$$

where: $$Q_{eq} \equiv \frac{|X|}{R} \cong \left[\frac{1}{Q_s} + \frac{1}{Q_p}\right]^{-1} \qquad Q_s \equiv \frac{\omega L}{R_s} \qquad Q_p \equiv \frac{R_p}{\omega L} \quad (1.12)$$

Q_{eq} is the global quality factor including the losses in the core represented by Q_p and the losses in the windings represented by Q_s .

HYSTERESIS LOSSES (2/2)

Eqn. 1.12 shows that the global quality factor Q_{eq} has a maximum as a function of frequency. In fact, the situation is more complicated, because the losses in the core and in the windings tend to increase with frequency. Typical quality factor curves as a function of frequency for iron-powder toroidal cores are shown in Fig. 1-14.

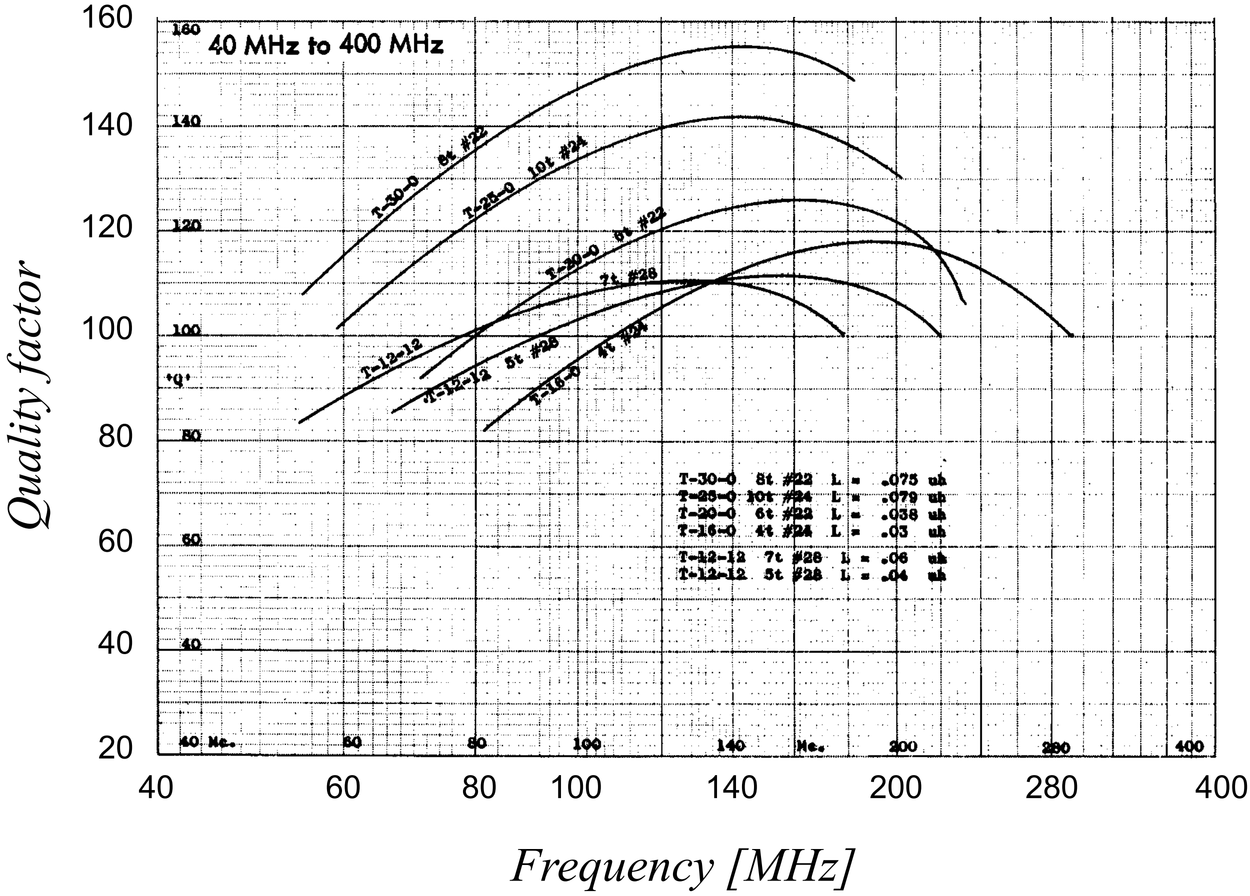


Fig 1-14: *Quality factors for iron-powder toroidal cores.*

TOROIDAL INDUCTOR DESIGN

The inductance of a magnetic-core toroidal inductor functioning in the linear part of its magnetization curve is approximately given by:

$$L = \frac{0.4\pi \cdot \mu_{init} \cdot A_c \times 10^{-2}}{l_e} \cdot N^2 = A_L \times 10^3 \cdot N^2 \qquad [\mu H] \quad (1.13)$$

where L is the inductance in μH, μ_{init} the initial permeability, A_c the cross-section of the core in cm^2, l_e the effective length of the core in cm and N the number of turns. A_L represents the inductance in nH for one single turn. The manufacturer usually specifies the factor A_L , which for a given inductance allows us to calculate the number of turns:

$$N = \sqrt{L / A_L} \qquad (1.14)$$

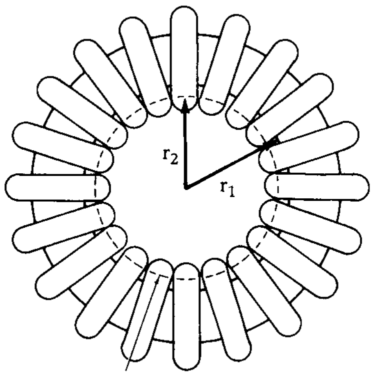


Fig 1-15: *Magnetic-core toroidal inductor.*

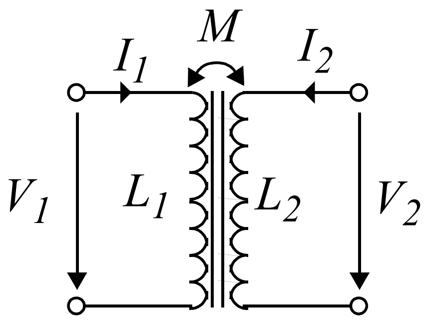
The inner radius of the toroid r_1 is also given by the manufacturer. From Fig. 1-15, we obtain the diameter of the wire:

$$d \le \frac{2\pi r_1}{N + \pi} \qquad (1.15)$$

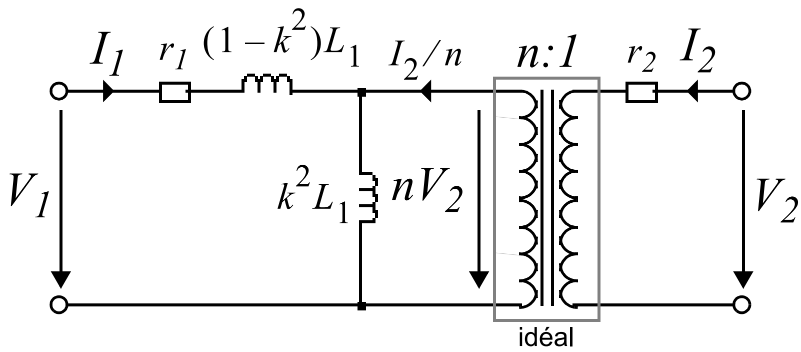
It is best to include a margin of 10% to take the possible variation in wire diameter into consideration.

TRANSFORMERS (1/2)

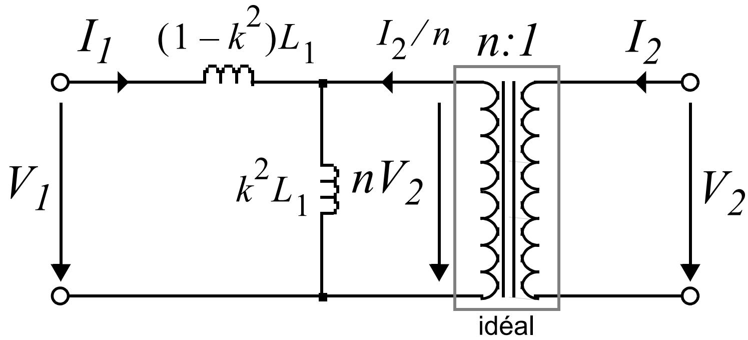
Transformers are used for coupling resonant circuits, phase inversion, galvanic isolation, and impedance matching. Since the transformer already contains an inductor, we can just add a primary or (and) secondary capacitor in order to make a (two) resonant (coupled) circuits. The equivalent circuit for a transformer is shown in Fig. 1-16 b). When the series resistances r_1 and r_2 representing the losses are much smaller than the source and load impedances, they can be ignored and the equivalent circuit simplifies to that of Fig. 1-16 c).



a) Transformer



b) Equivalent circuit.



c) With negligeable losses.

Fig 1-16: *Equivalent circuit of transformer.*

The equations corresponding to the circuit in Fig. 1-16 a) are:

$$\begin{aligned} V_1 &= sL_1 \cdot I_1 + sM \cdot I_2 \\ V_2 &= sM \cdot I_1 + sL_2 \cdot I_2 \end{aligned} \quad (1.16)$$

where M is the mutual inductance representing the coupling between the primary and the secondary.

TRANSFORMERS (2/2)

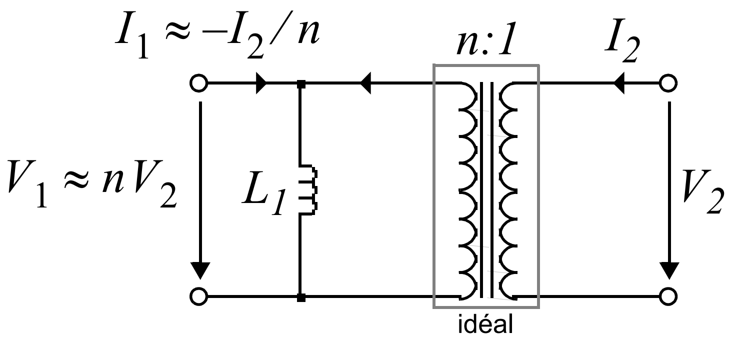
The equivalent circuit uses an ideal transformer with a transformation ratio n. The voltage at the primary of the ideal transformer is thus $n \cdot V_2$, while the current is divided by n. The equations of the equivalent circuit of Fig. 1-16 c) are given by:

$$V_1 = s(1-k^2)L_1 \cdot I_1 + sk^2 L_1 \cdot \left(I_1 + \frac{I_2}{n}\right) = sL_1 \cdot I_1 + \frac{sk^2 L_1 \cdot I_2}{n}$$
$$nV_2 = sk^2 L_1 \cdot \left(I_1 + \frac{I_2}{n}\right) \rightarrow V_2 = \frac{sk^2 L_1 \cdot I_1}{n} + \frac{sk^2 L_1 \cdot I_2}{n^2} \qquad (1.17)$$

By combining equations (1.16) and (1.17), we get:

$$\boxed{n = k \cdot \sqrt{\frac{L_1}{L_2}} \qquad k = \frac{M}{\sqrt{L_1 \cdot L_2}}} \qquad (1.18)$$

In the case where the coupling coefficient k is close to one, the equivalent circuit of Fig. 1.16 c) reduces to that of Fig. 1.17.



$$k \approx 1$$
$$M \approx \sqrt{L_1 \cdot L_2}$$
$$n \approx \sqrt{L_1/L_2} = N_1/N_2$$

Fig 1-17: *Equivalent circuit of transformer for $k \approx 1$.*

When a load R_L is connected to the secondary, it is seen by the primary as a load $n^2 R_L$ (cf Fig. 1.18).

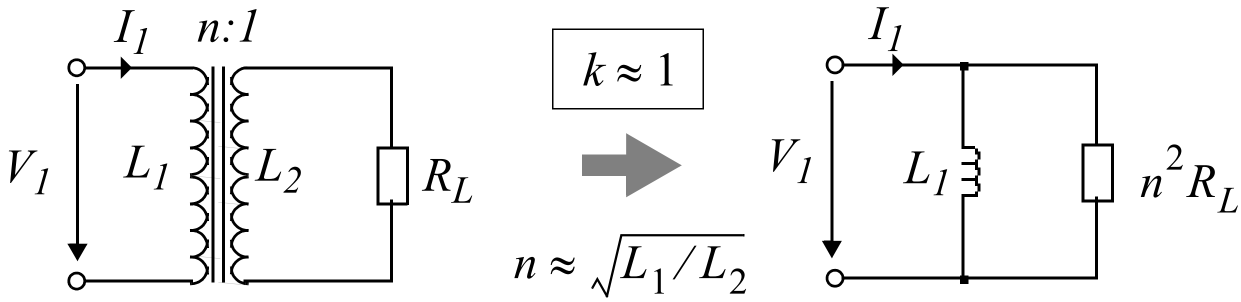
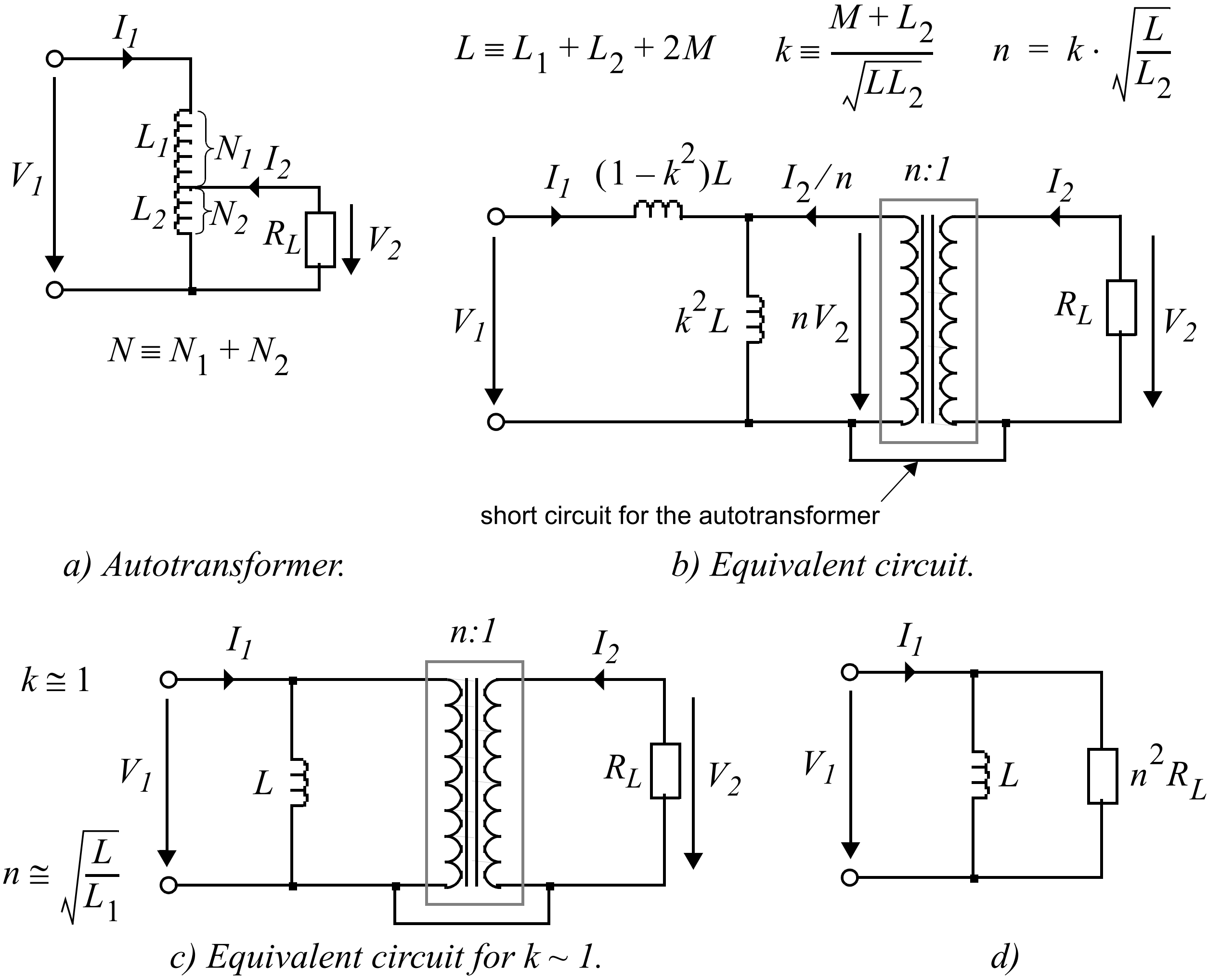


Fig 1-18: *Equivalent load as seen by the primary.*

AUTOTRANSFORMERS (1/2)

A transformer can often be advantageously replaced by an autotransformer, which is a coil of N turns having an intermediary point taken between N_1 and N_2 turns from the two ends (cf Fig. 1-19 a). The loop equations corresponding to the autotranformer of Fig. 1-19 a) are:

$$\begin{aligned} V_1 &= sL_1I_1 + sM(I_1+I_2) + sL_2(I_1+I_2) + sMI_1 \\ &= s(L_1+L_2+2M)I_1 + s(L_2+M)I_2 \\ V_2 &= sL_2(I_1+I_2) + sMI_1 = s(L_2+M)I_1 + sL_2I_2 \end{aligned} \tag{1.19}$$



a) Autotransformer. b) Equivalent circuit.

c) Equivalent circuit for k ~ 1. d)

Fig 1-19: *Autotransformer connected to a load.*

AUTOTRANSFORMERS (2/2)

The autotransformer can be replaced by an ideal transformer with a transformation ratio n of which the negative terminals of the primary and secondary are short-circuited. The resulting diagram is shown in Fig. 1-19 b) and the corresponding equations are given by:

$$
\begin{aligned}
V_1 &= s(1-k^2)LI_1 + nV_2 = sLI_1 + s\frac{k^2}{n}LI_2 \\
V_2 &= \frac{1}{n}\cdot\left(sk^2L\cdot\left(I_1+\frac{I_2}{n}\right)\right) = s\frac{k^2}{n}LI_1 + s\frac{k^2}{n^2}LI_2
\end{aligned}
\qquad (1.20)
$$

Combining Eqn. 1.20 with Eqn. 1.19, we find:

$$
L = L_1 + L_2 + 2M \qquad n = k\cdot\sqrt{\frac{L}{L_2}} \qquad (1.21)
$$

In the case where the coupling between the two coils is perfect ($k \cong 1$), the series inductance can be ignored and the diagram of Fig. 1-19 b) reduces to that of Fig. 1-19 c). Since the load impedance as seen from the primary is simply multiplied by the transformation ratio squared, the diagram of Fig. 1-19 c) can be further simplified to that of Fig. 1-19 d).
For $k \cong 1$, the transformation ratio and therefore the multiplication factor of the load impedance is given by:

$$
n = \sqrt{\frac{L}{L_2}} \qquad (1.22)
$$